15-251: Great Theoretical Ideas In Computer Science

Recitation 12

Rookie Mistake

A rook is placed in the lower left corner of an 8×8 chessboard. On each turn, the rook moves to a random legal location. The rook must move within its row or within its column and it cannot stay still. Let T denote the number of turns it takes until the rook lands in the upper right corner of the chessboard.

(a) Define the states and write the transition matrix for this Markov process.

(b) Find $\mathbf{E}[T]$.

No directions

Let G be an undirected graph and consider the following random process. Start at the vertex 1, and at each step, pick and then move to a random neighbor of the current vertex.

(a) What does the transition matrix look like for this Markov chain?

(b) What is the stationary distribution?

General Communication

Let $F : X \times Y \to Z$ be a communication function. (In class, we focused on the case $X = Y = \{0,1\}^n$ and $Z = \{0,1\}$). Show that $\mathbf{D}(F) \leq \lceil \log_2 |X| \rceil + \lceil \log_2 |Z| \rceil$

Rectangle

Let $F : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}$ be a communication function and let M_F be the associated function matrix. Suppose that every monochromatic rectangle in M_F has size (number of entries) at most 2^n. What can you conclude about $\mathbf{D}(F)$?

Sharing randomization

Recall the randomized communication complexity model introduced in class. There, the players were allowed to individually flip coins, and make decisions based on the outcomes of those coin flips. We showed that $\mathbf{R}^{\epsilon}(\text{EQ}) = O(\log n)$, where the error probability $\epsilon = 1/n$. In this question we'll consider a slightly different randomized communication complexity model. In this new model, we'll assume that the players share a public coin. Whenever one of the players flips this coin, the other player automatically sees the outcome of the coin flip (without them communicating any bits). Show that in this model, there is a randomized protocol for EQ of cost $O(1)$ and error probability $1/2^{300}$. What is the exact relationship between the cost and the error probability of your protocol?