15-251: Great Theoretical Ideas In Computer Science

Recitation 6

Announcements

- We have a midterm this Wednesday, October 14.

PRIMES is in P

Consider the following algorithm that determines if a given number is prime or not.

```
isPrime(N):
    if (N < 2):
        return False
    for factor in {2,3,4,...,N-1}:
        if (N % factor == 0):
            return False
    return True
```

(a) Assume that the input to the function is encoded in binary. What is the length of the input, in terms of N, using this encoding scheme?

(b) What is the running time (in Big-Oh) of the long division algorithm you have learned in grade school?

(c) What is the running time of the above function isPrime in Big-Oh as a function of the input length?

Decisions

Prove whether or not each of the following are decidable.

(a) $S = \{M \mid \text{there exists a circuit family that decides } \mathcal{L}(M)\}$

(b) $T = \{M \mid \mathcal{L}(M) \in \mathsf{P}\}$

Fun fact: The set of winning positions for white in generalized $n \times n$ chess is known to be decidable but is also known not to be in P.

Uncounting

Are the following sets countable?

(a) The set of directed trees

(b) The set of circuit families

(c) Bonus: The set of circuit families that decide regular languages over the alphabet $\{0, 1\}$

Sneaky structures (part 2)

Prove by induction that a graph with maximum degree up to k must be $(k+1)$-colorable.

Warning: induction on graphs can lead to subtle logical pitfalls if you aren't careful. Contrast this proof with the bad induction from recitation 1 and understand why the latter doesn't work.

Counting is hard

Define a function smash where smash(n) returns the in-order concatenation of all the numbers $0 - n$. For example, smash(10) = "012345678910". Show that $L = \{\text{smash}(n) \mid n \in \mathbb{N}\}$ is irregular.