CMU 15-251
Computational Social choice

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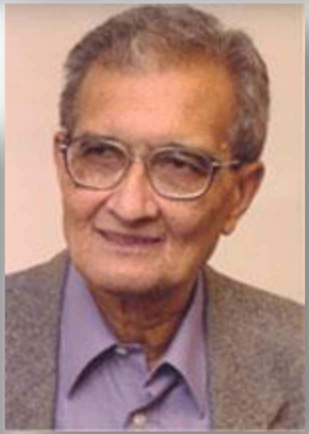
Social choice theory

- A mathematical theory that deals with aggregation of individual preferences
- Origins in ancient Greece
- Formal foundations: 18th Century (Condorcet and Borda)
- 19th Century: Charles Dodgson
- 20th Century: Nobel prizes to Arrow and Sen



The voting model

- Set of voters $N = \{1, \dots, n\}$
- Set of alternatives $A, |A| = m$
- Each voter has a ranking over the alternatives
- Preference profile = collection of all voters' rankings

1	2	3
a	c	b
b	a	c
c	b	a

Voting rules

- Voting rule = function from preference profiles to alternatives that specifies the winner of the election
- Plurality
  - Each voter awards one point to top alternative
  - Alternative with most points wins
  - Used in almost all political elections

More voting rules

- Borda count
  - Each voter awards $m - k$ points to alternative ranked k'th
  - Alternative with most points wins
  - Proposed in the 18th Century by the chevalier de Borda
  - Used for elections to the national assembly of Slovenia
  - Similar to rule used in the Eurovision song contest

Lordi, Eurovision 2006 winners

More voting rules

- x beats y in a pairwise election if the majority of voters prefer x to y
- Plurality with runoff
  - First round: two alternatives with highest plurality scores survive
  - Second round: pairwise election between these two alternatives

More voting rules

- Single Transferable vote (STV)
  - $m - 1$ rounds
  - In each round, alternative with least plurality votes is eliminated
  - Alternative left standing is the winner
  - Used in Ireland, Malta, Australia, and New Zealand (and Cambridge, MA)

STV: EXAMPLE

2 voters	2 voters	1 voter
a	b	c
b	a	d
c	d	b
d	c	a

2 voters	2 voters	1 voter
a	b	c
b	a	b
c	c	a

2 voters	2 voters	1 voter
a	b	b
b	a	a

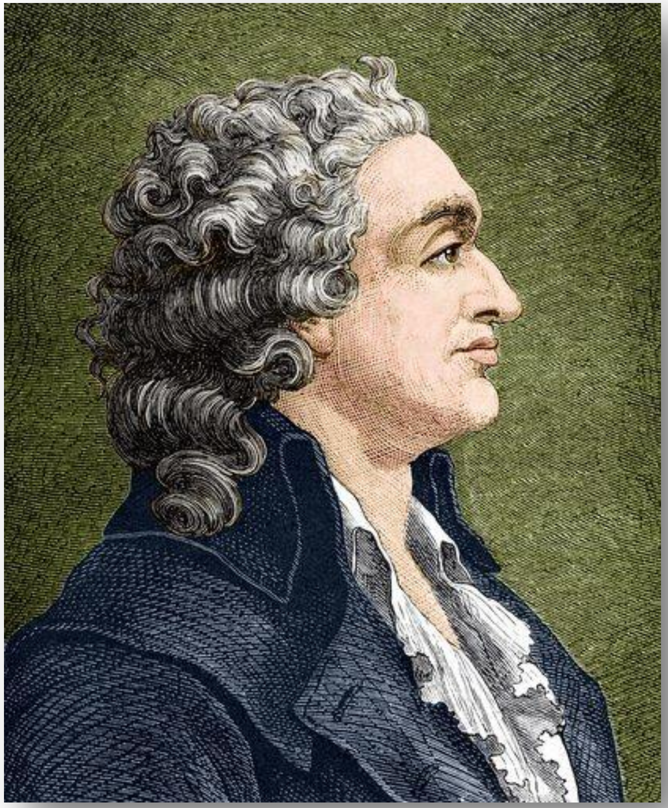
2 voters	2 voters	1 voter
b	b	b

Social choice axioms

- How do we choose among the different voting rules? Via desirable properties!
- Majority consistency = if a majority of voters rank alternative x first, then x should be the winner
- Vote: Which rule is not majority consistent?
  1. Plurality
  2. Plurality with runoff
  3. Borda count
  4. STV

Marquis de Condorcet

- 18th Century French Mathematician, philosopher, political scientist
- One of the leaders of the French revolution
- After the revolution became a fugitive
- His cover was blown and he died mysteriously in prison



Condorcet winner

- Recall: x beats y in a pairwise election if a majority of voters rank x above y
- Condorcet winner beats every other alternative in pairwise election
- Condorcet paradox = cycle in majority preferences

1	2	3
a	c	b
b	a	c
c	b	a

Condorcet consistency

- Condorcet consistency = select a Condorcet winner if one exists
- Vote: Which rule is Condorcet consistent?
  1. Plurality
  2. Borda count
  3. Both
  4. Neither

Condorcet consistency

- Vote: What is the relation between majority consistency and Condorcet consistency?
  1. Majority cons. $\Rightarrow$ Condorcet cons.
  2. Condorcet cons. $\Rightarrow$ Majority cons.
  3. Equivalent
  4. Incomparable

More voting rules

- Copeland: Alternative's score is #alternatives it beats in pairwise elections
- Why does Copeland satisfy the Condorcet criterion?
  - If x is a Condorcet winner, score $= m - 1$
  - Otherwise, score $< m - 1$

Awesome example

- Plurality: *a*
- Borda: *b*
- Condorcet winner: *c*
- STV: *d*
- Plurality with runoff: *e*

33 voters	16 voters	3 voters	8 voters	18 voters	22 voters
a	b	c	c	d	e
b	d	d	e	e	c
c	c	b	b	c	b
d	e	a	d	b	d
e	a	e	a	a	a

Manipulation

- Using Borda count
- Top profile: *b* wins
- Bottom profile: *a* wins
- By changing his vote, voter 3 achieves a better outcome!
- Borda responded: “My scheme is intended only for honest men!”

1	2	3
b	b	a
a	a	b
c	c	c
d	d	d

1	2	3
b	b	a
a	a	c
c	c	d
d	d	b

Strategyproofness

- A voting rule is strategyproof (SP) if a voter can never benefit from lying about his preferences
- Vote: What is the largest value of m for which plurality is SP?
  1. $m = 1$
  2. $m = 2$
  3. $m = 3$
  4. $m = \infty$

Gibbard-Satterthwaite

- A voting rule is dictatorial if there is a voter who always gets his most preferred alternative
- A voting rule is onto if any alternative can win
- **Theorem (Gibbard-Satterthwaite)**: If $m \geq 3$ then any voting rule that is SP and onto is dictatorial
- In other words, any voting rule that is onto and nondictatorial is manipulable

Complexity of manipulation

- Manipulation is always possible in theory
- But can we design voting rules where it is difficult in practice?
- Are there "reasonable" voting rules where manipulation is a hard computational problem? [Bartholdi et al., SC&W 1989]

The computational problem

- *R*-Manipulation problem:
  - Given votes of nonmanipulators and a preferred candidate p
  - Can manipulator cast vote that makes p uniquely win under R?
- Example: Borda, $p = a$

1	2	3
b	b	
a	a	
c	c	
d	d	

1	2	3
b	b	a
a	a	c
c	c	d
d	d	b

A greedy algorithm

- Rank p in first place
- While there are unranked alternatives:
  - If there is an alternative that can be placed in next spot without preventing p from winning, place this alternative
  - Otherwise return false

Example: Borda

1	2	3
b	b	a
a	a	
c	c	
d	d	

1	2	3
b	b	a
a	a	b
c	c	
d	d	

1	2	3
b	b	a
a	a	c
c	c	
d	d	

1	2	3
b	b	a
a	a	c
c	c	b
d	d	

1	2	3
b	b	a
a	a	c
c	c	d
d	d	

1	2	3
b	b	a
a	a	c
c	c	d
d	d	b

When does the alg work?

- **Fact:** The greedy algorithm is a polynomial-time algorithm for plurality, Borda count, plurality with runoff, Copeland, and Maximin
- **Theorem** [Bartholdi and Orlin, 1991]: the STV-Manipulation problem is NP-complete!

What we have learned

- Definitions:
  - Plurality, Borda count, plurality with runoff, STV, Copeland, Maximin
  - Majority consistency
  - Condorcet winner, Condorcet consistency
  - Strategyproofness
  - The Gibbard-Satterthwaite Thm
- Principles:
  - NP-hardness can be good!